MTH 322

1. $f(t) = \sin^3(t)$

$L \sin^3(t) = L(\sin t(\sin^2 t)$ but $\sin^2 t = 1 - \cos^2 t$

$L \sin^3 t = L \sin t(1-\cos^2 t) = L \sin t - \sin t \cos^2 t$

but $\cos^2 t = \frac{1}{2}[1+\cos 2t]$

$L^{-1} \sin^3 t = L[\sin t - \sin t[\frac{1}{2} + \frac{1}{2}\cos 2t]]$

$= L \sin t - \frac{1}{2}\sin t - \frac{1}{2}\sin t \cos 2t.$

but $\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$L \sin^3 t = L\frac{1}{2}\sin t - \frac{1}{4}(\sin 3t + \sin -t)$

$L \sin^3 t = L\frac{1}{2}\sin t - \frac{1}{4}\sin 3t - \frac{1}{4}\sin -t$

$= \frac{1}{2}\left[\frac{1}{s^2+1}\right] - \frac{1}{4}\frac{3}{s^2+9} + \frac{1}{4}\frac{1}{s^2+1}$

$= \frac{1}{2}\left[\frac{1}{s^2+1}\right] + \frac{1}{4}\left[\frac{1}{s^2+1}\right] - \frac{3}{4}\frac{1}{s^2+9}$

$= \frac{3}{4}\left(\frac{1}{s^2+1}\right) - \frac{3}{4s^2+36}$

$= \frac{3}{4s^2+4} - \frac{3}{4s^2+36} = \frac{3}{4}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$

2. $\frac{4s^2+5s-3}{(s-1)(s+2)(s+1)}$

Using cover up rule

$\frac{4s^2+5s-3}{(s-1)(s+2)(s+1)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s+1}$

$A = \frac{(s-1)(4s^2+5s-3)}{(s-1)(s+2)(s+1)} = \frac{A}{(s-1)} + \frac{B(s-1)}{s+2} + \frac{C(s-1)}{s+1} \Big|_{s=1}$

$A = \frac{4s^2+5s-3}{(s+2)(s+1)}\Big|_{s=1} = \frac{4+5-3}{3(2)} = \frac{6}{6} = 1$

$$B = \frac{4s^2+5s-3}{(s-1)\cancel{(s+2)}(s+1)}\cancel{(s+2)} = \frac{A}{(s-1)}(s+2) + \frac{B}{\cancel{(s+2)}}\cancel{(s+2)} + \frac{C}{(s+1)}(s+2) \Bigg|_{s=-2}$$

$$B = \frac{4s^2+5s-3}{(s-1)(s+1)} \Bigg|_{s=-2}$$

$$B = \frac{16+(-10)-3}{(-3)(-1)} = \frac{3}{3} = 1$$

$$C = \frac{(4s^2+5s-3)\cancel{(s+1)}}{(s-1)(s+2)\cancel{(s+1)}} = \frac{A}{s-1}(s+1) + \frac{B}{s+2}(s+1) + \frac{C}{\cancel{s+1}}\cancel{(s+1)} \Bigg|_{s=-1}$$

$$C = \frac{4s^2+5s-3}{(s-1)(s+2)} \Bigg|_{s=-1}$$

$$C = \frac{4-5-3}{(-2)(1)} = \frac{-4}{-2} = 2$$

$$L\ \frac{4s^2+5s-3}{(s-1)(s+2)(s+1)} = \frac{1}{s-1} + \frac{1}{s+2} + \frac{2}{s+1}$$

$$= e^{+t} + e^{-2t} + 2e^{-t}$$

$$= e^{t} + e^{-2t} + 2e^{-t}$$

(b) $$\frac{s^2-s+4}{(s^2-2s+2)(s+2)} = \frac{A}{s^2-2s+2} + \frac{B}{s+2}$$

completing the square

$$= \frac{A}{s^2-(2s+2-(\frac{2}{2})^2+1} + \frac{B}{s+2}$$

$$= \frac{A}{s^2-2s+1+1} + \frac{B}{s+2}$$

$$= \frac{A}{(s-1)^2+1} + \frac{B}{s+2}$$

$$\frac{s^2 - s + 4}{(s^2 - 2s + 2)\cancel{(s+2)}}\cancel{(s+2)} = \frac{A}{(s-1)^2 + 1}(s+2) + \frac{B}{\cancel{(s+2)}}(s+2) \Bigg|_{s=-2}$$

$$B = \frac{s^2 - s + 4}{s^2 - 2s + 2}\Bigg|_{s=-2}$$

$$B = \frac{4 + 2 + 4}{4 + 4 + 2} = 1$$

$$\frac{s^2 - s + 4}{\cancel{(s^2 - 2s + 2)}(s+2)}\cancel{(s-1)^2+1} = \frac{A\cancel{(s-1)^2+1}}{\cancel{(s-1)^2+1}} + \frac{B}{(s+2)}\left((s-1)^2 + 1\right) \Bigg|_{s=1}$$

$$A = \frac{s^2 - s + 4}{s + 2}\Bigg|_{s=1}$$

$$A = \frac{1 - 1 + 4}{3} = \frac{4}{3}$$

$$\mathcal{L}^{-1}\frac{s^2 - s + 4}{(s^2 - 2s + 2)(s+2)} = \mathcal{L}^{-1}\left[\frac{4/3}{(s-1)^2 + 1} + \frac{1}{s+2}\right]$$

$$= \mathcal{L}^{-1}\frac{4}{3}\left[\frac{1}{(s-1)^2 + 1}\right] + \frac{1}{s+2}$$

$$= \frac{4}{3}e^{-t}\sin t + e^{-t}$$

c) $$\frac{3s^2 - s}{\cancel{(s-1)^2}(s+1)}\cancel{(s-1)^2} = \frac{A}{\cancel{(s-1)^2}}\cancel{(s-1)^2} + \frac{B}{s+1}(s+1)^2 \Bigg|_{s=1}$$ cover up rule.

$$A = \frac{3s^2 - s}{s + 1}\Bigg|_{s=1} = \frac{3 - 1}{2} = \frac{2}{2} = 1$$

$$B = \frac{3s^2 - s}{(s-1)^2}\Bigg|_{s=-1} = \frac{3 + 1}{4} = 1$$

$$\mathcal{L}^{-1}\frac{3s^2-s}{(s-1)^2(s+1)} = \mathcal{L}^{-1}\frac{1}{(s-1)^2} + \mathcal{L}^{-1}\frac{1}{s+1}$$

$$= te^{t} + e^{-t}$$